# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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No. 1158



## SOME AERODYNAMIC RELATIONS FOR AN AIRFOIL IN OBLIQUE FLOW

By F. Ringleb

Translation

"Einige aerodynamische Beziehungen für den Tragflügel bei Schräganströmung"

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# SOME AERODYNAMIC RELATIONS FOR AN AIRFOIL IN OBLIQUE FLOW*

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Abstract: Some aerodynamic relations are derived which exist between two infinitely long airfoils if one is in a straight flow and the other in oblique flow, and both present the same profile in the direction of flow.

1. To begin with, consider an infinitely long airfoil whose axis is parallel to the z-axis of a rectangular x, y, z coordinate system. (See fig. 1.) The flow takes place with the velocity $V_o$ at infinity, parallel to the x, z plane and at the angle $\beta$ to the negative x-direction. The components of the velocity at infinity are

$$\left.\begin{aligned} u_o &= -V_o \cos \beta \\ v_o &= 0 \\ w_o &= V_o \sin \beta \end{aligned}\right\} \qquad (1)$$

For straight flow $\beta = 0$. The velocity potential of such a flow, which depends on $x$ and $y$, solely, can be written in the form

$$\Phi(x, y) = -u_o x + \Phi_o(x, y)$$

Consider now, the three-dimensional flow with the velocity potential

$$\Phi(x, y, z) = -u_o x + w_o z + \Phi_o(x, y)$$

*"Einige aerodynamische Beziehungen für den Tragflügel bei Schräganströmung," Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB), Berlin-Adlershof, FB 1497, Oct. 21, 1941.

The velocity components u, v, w at an arbitrary position x, y, z of space are consequently

$$u = \frac{\partial\Phi}{\partial x} = -u_o + \frac{\partial\Phi_o}{\partial x}$$

$$v = \frac{\partial\Phi}{\partial y} = \frac{\partial\Phi_o}{\partial y}$$

$$w = \frac{\partial\Phi}{\partial z} = w_o$$

The velocity components of the oblique flow with velocity $V_o$ are obtained by adding $w_o$ to the velocity components of the straight flow at every position of the components. The streamlines of the oblique flow result from the system of differential equations

$$dx: dy: dz = \left(-u_o + \frac{\partial\Phi_o}{\partial x}\right) : \frac{\partial\Phi_o}{\partial y} : w_o$$

The first of these equations contains only the variables x and y. This equation also determines the streamlines for $\beta = 0$, at the same time, therefore $w_o = 0$. Consequently, it is recognized that: <u>The projections of the streamlines of the oblique flow on the x, y plane are also the streamlines of the airfoil in the straight flow</u>. The three-dimensional flow defined by the potential $\Phi(x, y, z)$, therefore, furnishes a flow around the same airfoil as the straight flow determined by the potential $\Phi(x, y) = -u_o x + \Phi_o(x, y)$.

2. The same airfoil is to be considered in straight flow and then in oblique flow with the same velocity at infinity. To investigate the local change in the velocity in straight flow with flow velocity $u_o$ let u, v be the velocity components at any point in space. Adding on the component $w_o$ gives the oblique flow velocity $V_o$ and locally the velocity $V_s$, where

$$V_s^2 = u^2 + v^2 + w_o^2$$

In straight flow with $V_o$ instead of $u_o$ the local velocity components are $\frac{u}{\cos\beta}$, $\frac{v}{\cos\beta}$ and the velocity $V_g$ itself satisfies the equation

$$V_g^2 = \frac{u^2 + v^2}{\cos^2\beta}$$

Eliminating $u^2 + v^2$ from both relations furnishes

$$V_s^2 = v_o^2 + V_g^2 \cos^2\beta$$

Since

$$v_o = V_o \sin\beta$$

it therefore follows that: <u>If $V_g$ is the velocity at any position with straight flow past the airfoil with velocity $V_o$, and $V_s$ the velocity with oblique flow with the angle of obliquity $\beta$ and the same flow velocity $V_o$ then the following applies:</u>

$$\left(\frac{V_s}{V_o}\right)^2 = \sin^2\beta + \left(\frac{V_g}{V_o}\right)^2 \cos^2\beta \tag{2}$$

Figure 2 shows the velocity distributions for an 18-percent thickness profile with 30 percent rearward position of the maximum thickness for an angle of attack $\alpha = 0^\circ$ and the angles of obliquity $\beta = 0^\circ$ and $\beta = 45^\circ$.

For the pressure difference with respect to infinity in relation to the dynamic pressure

$$\left(\frac{\Delta p}{q}\right)_s = 1 - \left(\frac{V_s}{V_o}\right)^2$$

for oblique flow. It follows from this because of (2)

$$\left(\frac{\Delta p}{q}\right)_s = \left[1 - \left(\frac{V_g}{V_o}\right)^2\right] \cos^2\beta$$

therefore

$$\left(\frac{\Delta p}{q}\right)_s = \left(\frac{\Delta p}{q}\right)_g \cos^2\beta \tag{3}$$

where $\left(\frac{\Delta p}{q}\right)_g$ signifies the value of $\frac{\Delta p}{q}$ for straight flow. Consider an airfoil section parallel to the x, y plane and compare the $c_a$ and $c_m$ values for straight and oblique flow past the airfoil for the section profile at the same angle of attack.[1] If $V$ is the velocity at a profile point, generally

$$c_a = \frac{1}{l} \oint_L \left(\frac{V}{V_o}\right)^2 dx \tag{4}$$

$$c_m = -\frac{1}{l^2} \oint_L \left(\frac{V}{V_o}\right)^2 (x\ dx + y\ dy) \tag{5}$$

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[1]The angle of attack $\alpha$ does not vary with $\beta$.

for which the integrals are to extend over the profile contour in the positive sense. L is the length of the profile. The quantity $c_m$ is referred to the z-axis of the coordinate system. Applying both formulas to oblique flow and replacing $V_s$ by $V_g$ in accordance with (2), it follows that

$$c_{a_s} = \frac{1}{l} \int_L \left[\sin^2\beta + \left(\frac{V_g}{V_o}\right)^2 \cos^2\beta\right] dx$$

$$c_{m_s} = -\frac{1}{l^2} \int_L \left[\sin^2\beta + \left(\frac{V_g}{V_o}\right)^2 \cos^2\beta\right] (x\ dx + y\ dy)$$

But

$$\int_L dx = 0$$

$$\int_L (x\ dx + y\ dy) = 0$$

therefore

$$c_{a_s} = c_{a_g} \cos^2\beta \tag{6}$$

$$c_{m_s} = c_{m_g} \cos^2\beta \tag{7}$$

These are the relations between the lift and moment coefficients for straight and oblique flow past the same profile.[2]

[2] The preceding results are known, essentially though not derived in this form. See A. Busemann, Aerodynamischer Auftrieb bei Überschallgeschwindigkeit. Volta-Tagung 1935.

3. The relations between the lift and moment coefficients will be derived; these apply for two airfoils, one is in a straight, the other in an oblique flow and both present the same profile in the direction of flow. While the problem dealt with so far was of an elementary nature, insofar as the profile form of the airfoil had no effect on the result, the solution of the problem existing now requires more searching technique. The method of the conformal mean[3] developed elsewhere by the author is applied to the problem; the method is assumed to be known in this report. Consideration will be given now to the connection between the lift or moment coefficients of two profiles, one of which proceeds from the other by affine variation of the ordinates. Use will be made of the fact that the conformal mean between any profile $P_1$ that is not too thick and not too greatly cambered and the related straight section is affine to $P_1$ as an approximation. Figure 3 shows, for example, the symmetric hyperbolic profile with 18 percent of the maximum thickness and 30 percent rearward position of the maximum thickness and the 9-percent thickness profile that results from it by taking the conformal mean with the related section. The ordinates of the 9-percent thickness profile are made 20 times higher, those of the 18-percent thickness profile 10 times higher. It is seen that affinity almost prevails. The profile $P_1$ has length $l_1$ and thickness $d_1$, the related section has length $l_2$. The profile $P$, originating in the process of taking the conformal mean with the factor $\lambda = m:n$, has length $l$ and thickness $d$ (fig. 4). Setting

$$\frac{d}{l} = \delta$$

$$\frac{d_1}{l_1} = \delta_1$$

then[4]

$$\delta = \frac{\delta_1}{1 + \lambda\frac{l_2}{l_1}} \qquad (8)$$

[3]F. Ringleb - "Beiträge zur Profilsystematik" - ZWB Nr.
[4]See "Profilsystematik," Equation (31).

For assigned values of $\delta_1$, $\frac{l_2}{l_1}$, and $\delta$ it follows that

$$\lambda = \frac{l_1}{l_2}\left(\frac{\delta_1}{\delta} - 1\right) \tag{9}$$

With the same angle of attack $\alpha$ let $c_{a_1}$ be the lift coefficient of the profile $\underline{P_1}$ and $c_{a_2}$ the lift coefficient of the straight section. The lift coefficient $c_a$ of the profile $\underline{P}$ at the angle of attack $\alpha$ satisfies the relation[5]

$$\frac{1}{c_a} = \frac{1}{1 + \lambda}\left(\frac{1}{c_{a_1}} + \frac{\lambda}{c_{a_2}}\right)$$

Correspondingly, for the moment coefficients referred to the neutral point[6]

$$c_{m_o} = (1 + \lambda)\frac{{l_1}^2 c_{m_{o_1}} + \lambda {l_2}^2 c_{m_{o_2}}}{\left(l_1 + \lambda l_2\right)^2}$$

Now for the straight section

$$c_{a_2} = 2\pi \sin \alpha$$

$$c_{m_{o_2}} = 0$$

Accordingly,

$$\frac{1}{c_a} = \frac{1}{1 + \lambda}\left(\frac{1}{c_{a_1}} + \frac{\lambda}{2\pi \sin \alpha}\right)$$

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[5] See "Profilsystematik," Equation (38).
[6] See "Profilsystematik," Equation (40).

and since

$$c_{a_1} = 2\pi \frac{l_2}{l_1} \sin \alpha$$

the following is obtained

$$c_a = \frac{1 + \lambda}{\left(1 + \lambda \frac{l_2}{l_1}\right)} c_{a_1} \tag{10}$$

For $c_{m_o}$

$$c_{m_o} = \frac{1 + \lambda}{1 + \lambda\left(\frac{l_2}{l_1}\right)^2} c_{m_{o_1}} \tag{11}$$

Formulas (10) and (11) therefore give the connection sought between the lift or moment coefficients of two profiles which differ as a result of the affine variation of the ordinates; here $\lambda$ is defined by equation (9). It follows from (10) and (11):

$$\frac{c_{m_o}}{c_a} = \frac{1}{1 + \lambda \frac{l_2}{l_1}} \frac{c_{m_{o_1}}}{c_{a_1}}$$

and substituting for $\lambda$ the value from equation (9),

$$\frac{c_{m_o}}{c_a} = \frac{c_{m_{o_1}}}{c_{a_1}} \frac{\delta}{\delta_1} \tag{12}$$

This result indicates: With affine increase of the ordinates of a profile the distance of the lift vector from the neutral point increases, therefore, the distance of the center of pressure from the neutral point is proportional to the thickness of the profile. Consider two airfoils which present the same profile $P_1$ in the direction of flow, although one is in straight and the other in oblique flow. (See fig. 4.) The airfoil in oblique flow, therefore, presents a profile $P'$ in a perpendicular section, which is affinely thickened with respect to $P_1$. If $\beta$ is the angle of obliquity, then

$$\frac{\delta_1}{\delta'} = \cos\beta$$

If the profile $P'$ is in straight flow, the relations (10) and (11) hold with the values for $\lambda$ from (9). However, since $P'$ is in oblique flow at an angle $\beta$, these values for $c_a'$ and $c_{m_o}'$ from equations (10) and (11) must be multiplied by $\cos^2\beta$. Therefore

$$c_a' = \frac{1 + \lambda}{1 + \lambda\frac{l_2}{l_1}} c_{a_1} \cos^2\beta$$

$$c_{m_o}' = \frac{1 + \lambda}{\left(1 + \lambda\frac{l_2}{l_1}\right)^2} c_{m_{o_1}} \cos^2\beta$$

$$\lambda = \frac{l_1}{l_2}(\cos\beta - 1)$$

Accordingly

$$c_a' = \left[1 + \frac{l_1}{l_2}(\cos\beta - 1)\right] \cos\beta\, c_{a_1} \tag{13}$$

$$c_{m_o}' = \left[1 + \frac{l_1}{l_2}(\cos\beta - 1)\right] c_{m_{o_1}} \tag{14}$$

In these equations, therefore, $c_{a_1}$ and $c_{m_{o_1}}$ refer to the section perpendicular to the straight airfoil, $c_a'$ and $c_{m_o}'$ to the section perpendicular to the airfoil in oblique flow both of which present the same profile $P_1$ in the direction of flow. Since $c_{a_1} = 2\pi\frac{l_2}{l_1}\sin\alpha$ (13) can be written as

$$c_a' = 2\pi\left(\frac{l_2}{l_1} - 1 + \cos\beta\right)\sin\alpha\cos\beta \tag{15}$$

Forming the quotient of (14) and (13) gives

$$\frac{c_{m_o}'}{c_a'} = \frac{c_{m_{o_1}}}{c_{a_1}}\frac{1}{\cos\beta} \tag{16}$$

Introducing in place of the coefficients, the lift $A'$ and the moment $M_o'$

$$\frac{M_o'}{A'} = \frac{c_{m_o}'}{c_a'}l'$$

$$\frac{M_{o_1}}{A_1} = \frac{c_{m_{o_1}}}{c_{a_1}}l_1$$

and since $l' = l_1\cos\beta$

$$\frac{M_o'}{A'} \quad \frac{M_{o_1}}{A_1}$$

This result can be expressed, in brief, thusly: In the present affine transformation of the airfoil in straight flow to one in oblique flow, the position of the center of pressure of the section perpendicular to the airfoil axis experiences the same affine transformation. Let N be the neutral axis (position of the neutral point), D the position of the center of pressure of the airfoil in oblique flow, both of which are parallel to the airfoil edge. Further, let $a'$ be the distance of the center of pressure of the profile $\underline{P'}$ from its neutral point, correspondingly, let $a$ be the distance from the center of pressure of the profile $\underline{P}$ from its neutral point. Let $A'$ represent the lift of a strip of the airfoil surface parallel to the profile $\underline{P'}$ of unit width and $A$ the lift of a strip of the airfoil surface parallel to $\underline{P}$ with the same surface area (fig. 5), then

$$A = A'$$

therefore

$$c_a = c_a' \tag{17}$$

and for the related moments $M_o$ and $M_o'$ referred to the axes perpendicular to the profile planes and passing through the neutral points

$$M_o = aA$$

$$M_o' = a'A$$

Now (see fig. 5)

$$a' = \frac{a}{\cos \beta}$$

therefore

$$M_o' = \frac{M_o}{\cos \beta}$$

Since

$$M_o = c_{m_o} q l^2$$

$$M_o' = c_{m_o}' q l \, l'$$

$$l' = \frac{l}{\cos \beta}$$

and

$$c_{m_o} = c_{m_o}' \tag{18}$$

Together with equations (13) and (14) this gives the following result: If $c_a$ and $c_{m_o}$ are the coefficients of the lift and the moment for the profile (lying in the direction of flow) of an airfoil in oblique flow and $c_{a_1}$, $c_{m_{o_1}}$ the corresponding values for the same profile of an airfoil in straight flow, then

$$c_a = \left[1 - \frac{l_1}{l_2}(\cos \beta)\right] \cos \beta c_{a_1} \tag{19}$$

$$c_{m_o} = \left[1 - \frac{l_1}{l_2}(1 - \cos \beta)\right] c_{m_{o_1}} \tag{20}$$

where $\beta$ is the angle of obliquity and $\frac{l_1}{l_2}$ a profile constant. $\frac{l_1}{l_2}$ is the ratio of the profile length $l_1$ to the length $l_2$ of the section originating in its conformal transformation, which is slightly under one for the usual thin profiles. For various values of $\frac{d_1}{l_1}$ of the profile, the following values of the constant $\frac{l_1}{l_2}$

are obtained as approximations independently of the position of maximum thickness, the radius of the nose and the camber:

| | | | |
|---|---|---|---|
| $\frac{d}{l_1}$ | 0.18 | 0.09 | 0.00 |
| $\frac{l_1}{l_2}$ | 0.870 | 0.935 | 1.000 |

$\frac{l_1}{l_2}$ varies approximately linearly with the thickness. Expressed mathematically

$$\frac{l_1}{l_2} = 1 - 0.723\frac{d_1}{l_1}$$

From (19) and (20) the lift and moment coefficients (referred to the axis normal to the profile through its neutral point) for a section of the airfoil of unit width parallel to the profile. (See fig. 6.) For the airfoil $T_1$

$$A_1 = c_{a_1} l_1 q$$

$$M_1 = c_{m_{o_1}} {l_1}^2 q$$

and for the airfoil $T$

$$A = c_a \frac{1}{\cos\beta} l_1 \cos\beta q$$

$$M = c_{m_o} \frac{1}{\cos\beta} l_1 \cos\beta l_1 q$$

therefore

$$A = \left[1 - \frac{l_1}{l_2}(1 - \cos\beta)\right] \cos\beta A_1$$

$$M = \left[1 - \frac{l_1}{l_2}(1 - \cos\beta)\right] M_1$$

In all previous arguments the angle of attack $\alpha$ of the airfoil in oblique flow was defined as the angle of attack of a perpendicular section of the airfoil with respect to the x, z plane (fig. 1). However, it is usual in practice to define the angle of attack of an airfoil in oblique flow as the angle of attack $\alpha_\beta$ of a section of the airfoil in the direction of flow with respect to the x, z plane. Then

$$\tan\alpha_\beta = \tan\alpha \cos\beta$$

For small angles of attack the previous results furnish the findings summarized below.

## SUMMARY

Two airfoils $T_1$ and $T$ are considered, both infinite in extent, the first of which is in a straight flow, the second in oblique flow (angle of obliquity = $\beta$). Both profiles presented the same profile $P_1$ in the direction of flow with maximum thickness $d_1$ and length $l_1$. $c_{a_1}$ and $c_{m_{o_1}}$ are the coefficients for the lift and moment of the profile $T_1$ in straight flow, $c_a$ and $c_{m_o}$ the corresponding coefficients for the airfoil $T$ in oblique flow, where $c_{m_{o_1}}$ and $c_{m_o}$ refer to an axis perpendicular to the plane of the profile through its neutral point. For the same angle of attack (measured in the direction of flow) as a good approximation

$$c_a = kc_{a_1}$$

$$c_{m_o} = kc_{m_{o_1}}$$

where

$$k = \cos \beta + 0.723\frac{d_1}{l_1}(1 - \cos \beta)$$

Therefore, it follows that

$$\frac{\partial c_a}{\partial \alpha} = k\frac{\partial c_{a_1}}{\partial \alpha}$$

$$\frac{c_{m_o}}{c_a} = \frac{c_{m_{o_1}}}{c_{a_1}}$$

Figures 7 and 8 give a comparison of the first of these theoretical results with a measurement which has been carried out by section L of the Messerschmitt Company at the aerodynamic institute at Aachen on assorted swept-back wings of aspect ratio $\Lambda = 6.25$ and a symmetrical 12-percent thickness profile. The experimental results represented in the curves of figure 7 were converted for an infinite aspect ratio for the sake of comparison with the theory (fig. 8).

Translated by Dave Feingold
National Advisory Committee
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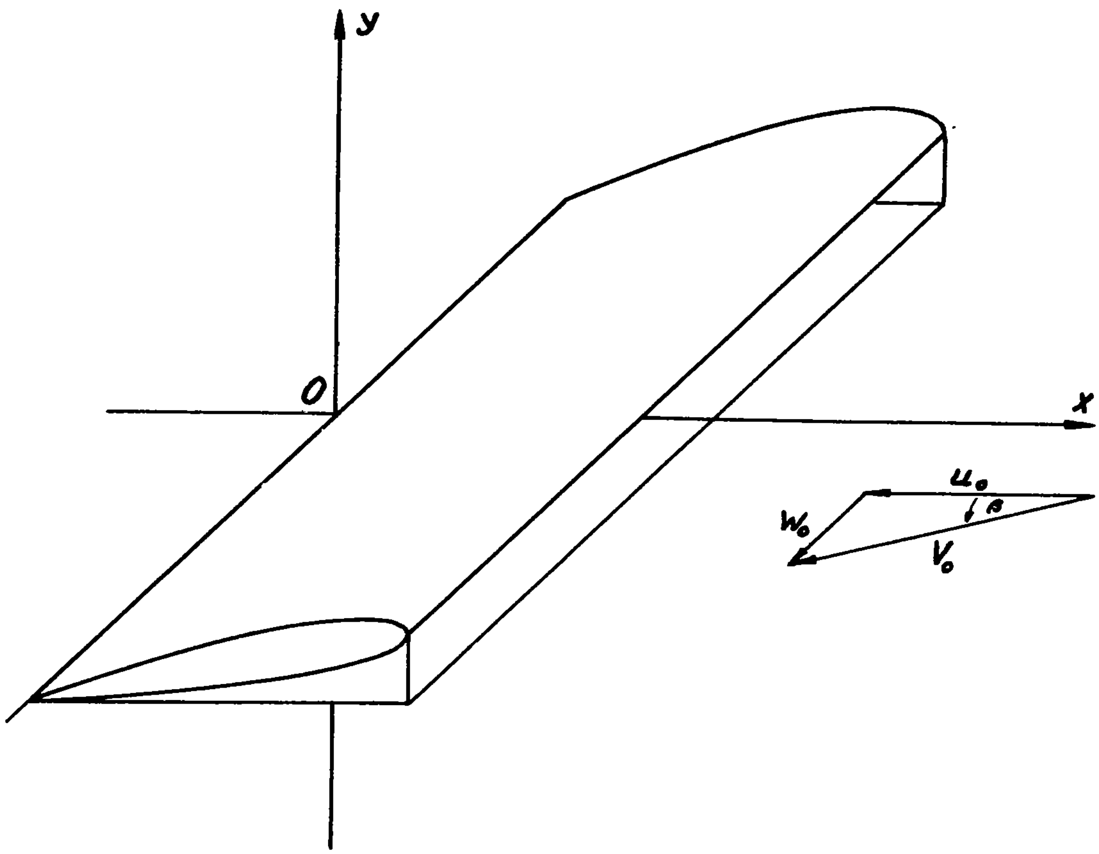



Figure 1.

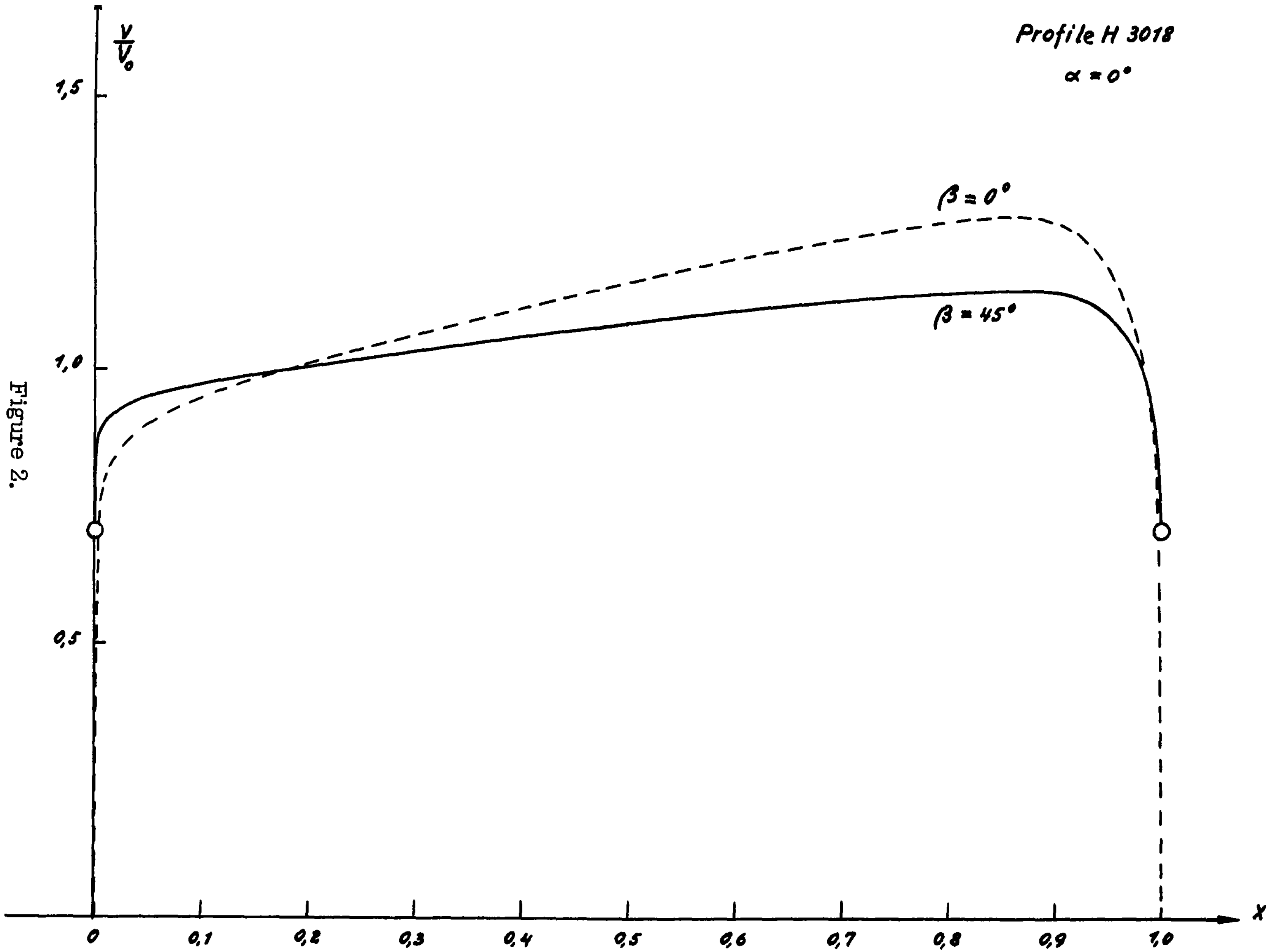
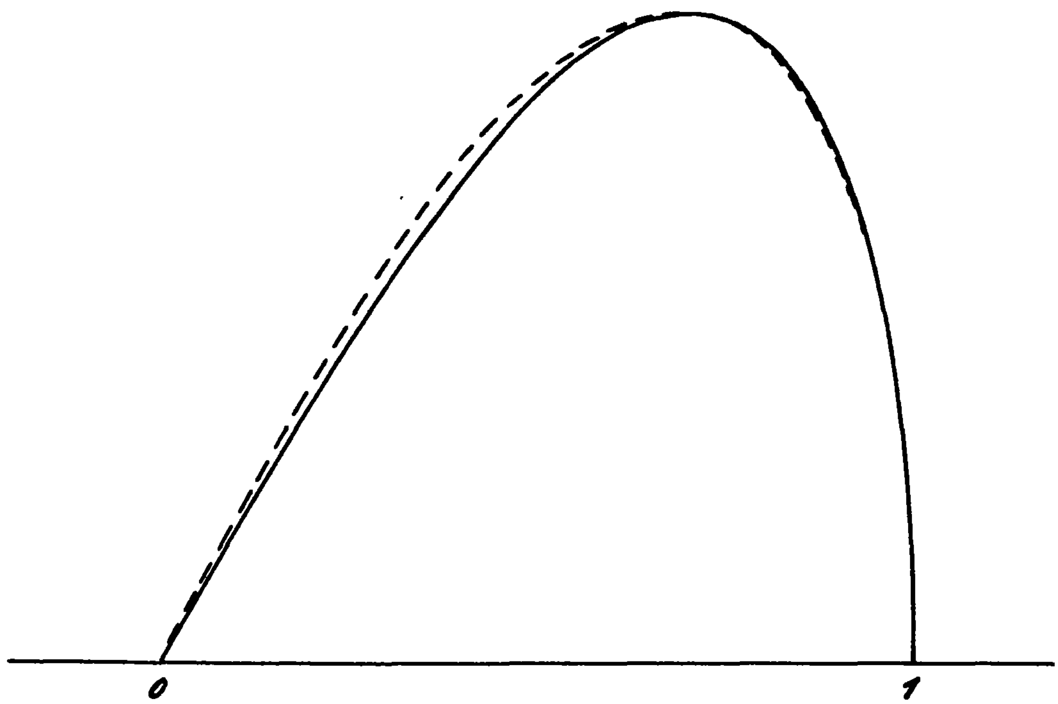

Profile H 3018
α = 0°
β = 0°
β = 45°
v/V₀
1,5
1,0
0,5
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
x

Figure 2.




Hyperbolic profile 3018 (made 10 x higher).

Hyperbolic profile 3018 and straight section (made 20 x higher).

Figure 3.

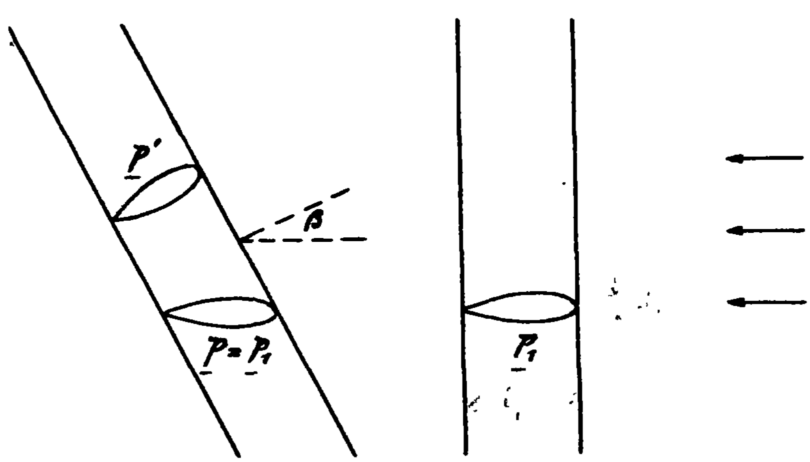



Figure 4.

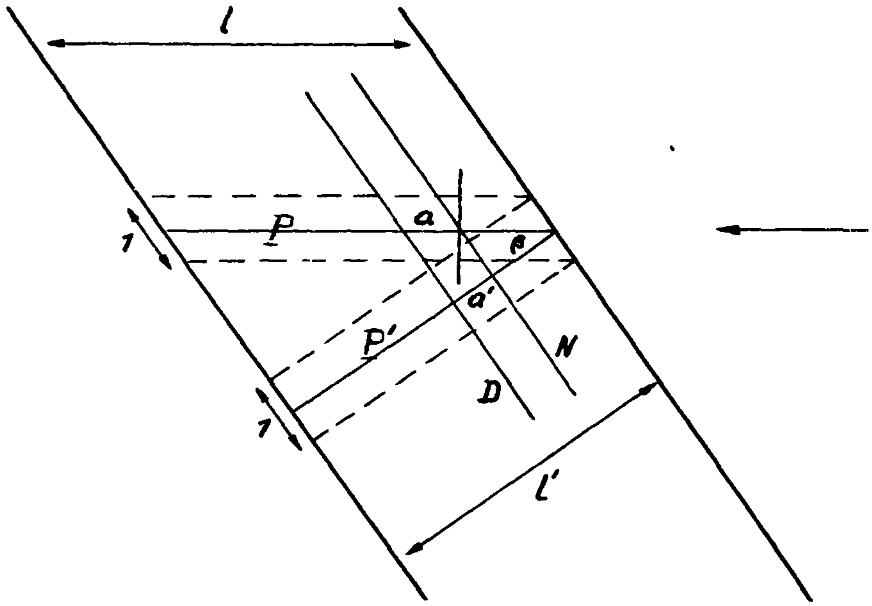



Figure 5.

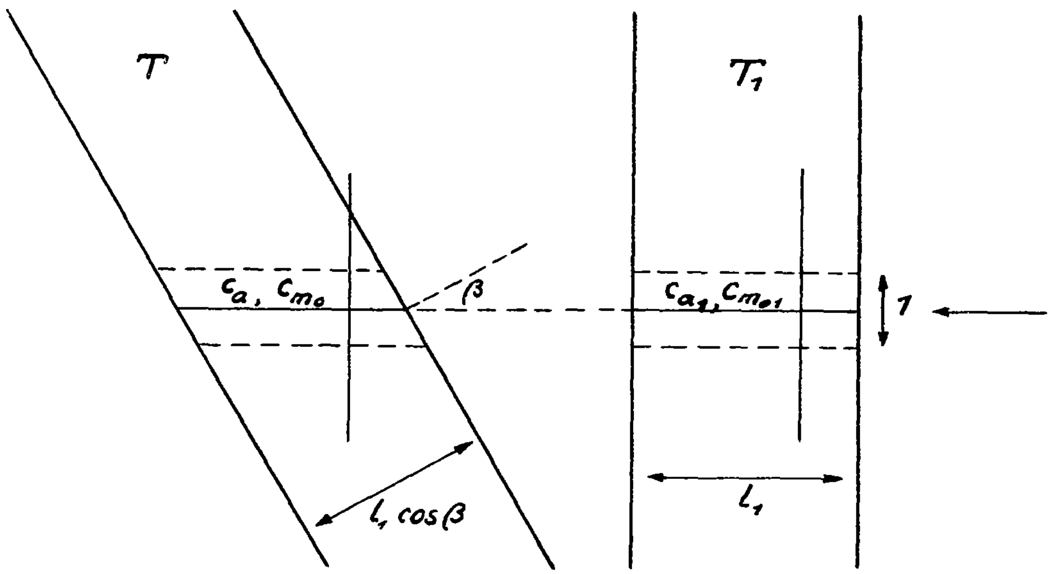



Figure 6.

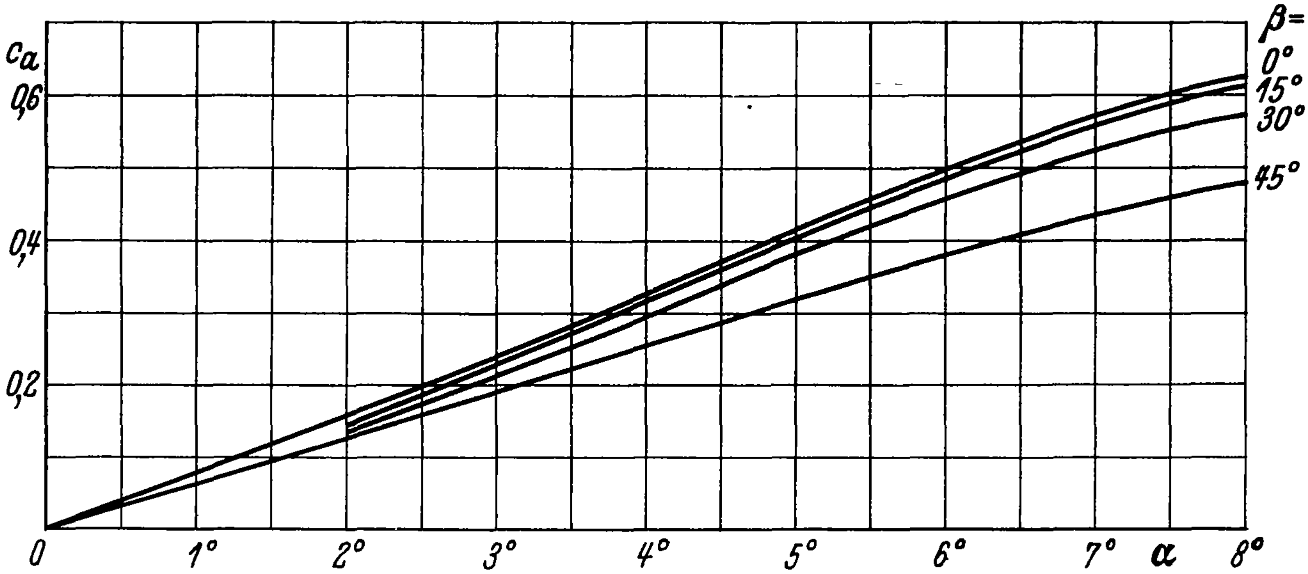



Figure 7.

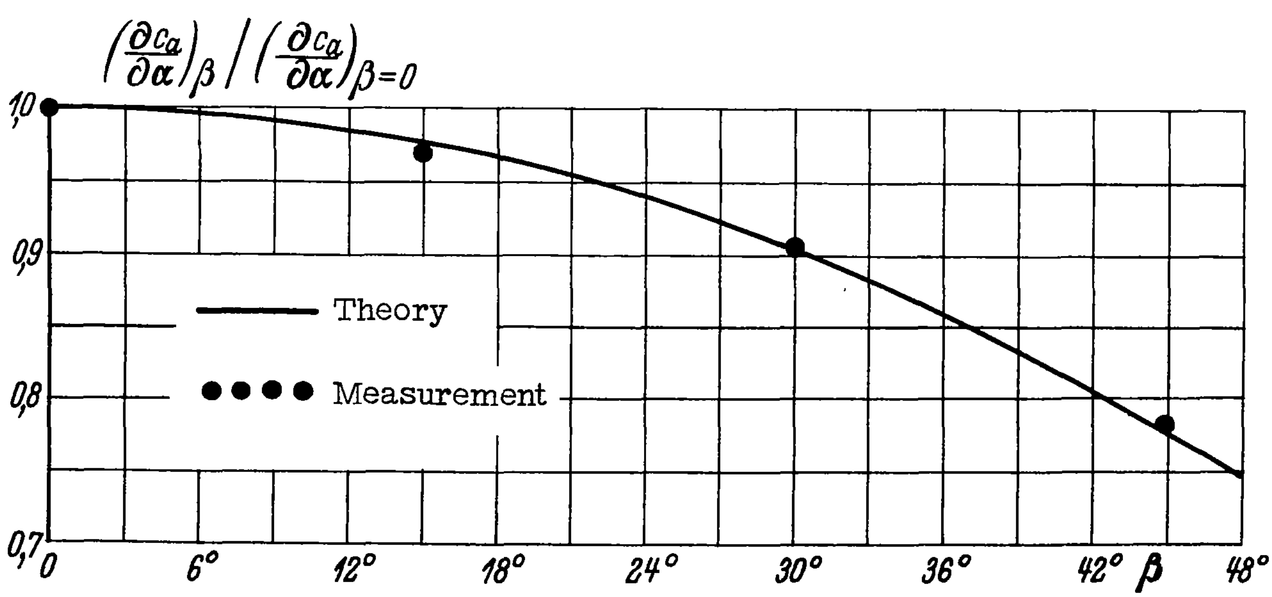



Figure 8.